

# Keeping Things In Proportion

exploring the structures underlying multiplicative relationships

[interwovenmaths.com/pti](https://interwovenmaths.com/pti)

---

**Nathan Day**

[@nathanday314](https://twitter.com/nathanday314)

[nathanday42@gmail.com](mailto:nathanday42@gmail.com)

[interwovenmaths.com](https://interwovenmaths.com)

# Aims

Do some maths

---

Try out different types of tasks

---

Think about connections

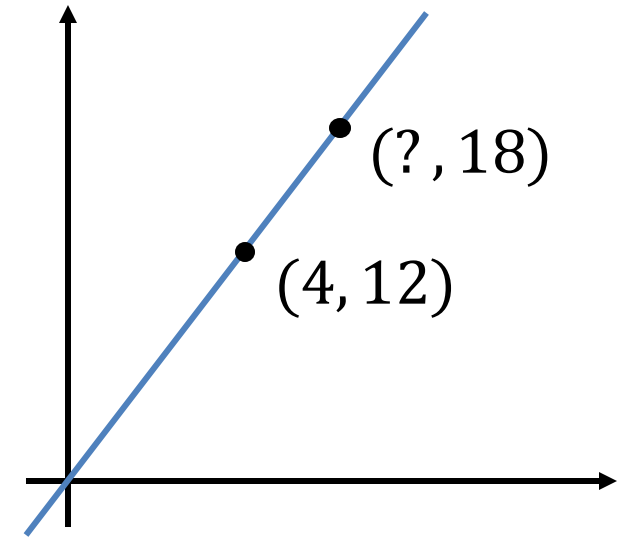
---

See some places to find resources

$$\frac{4}{12} = \frac{?}{18}$$

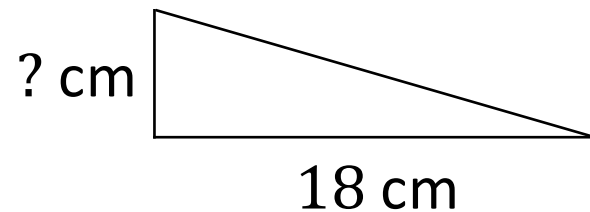
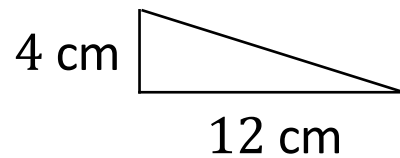
Kathryn earns £12 for every 4 trees she plants.

How much does she earn for planting 18 trees?



Mary walks 12 miles in 4 hours.

How long does it take her to walk 18 miles?



4 yards is equal to 12 feet.

How many yards are there in 18 feet?

Nikki spilled some ink over her maths homework.

Can you still answer the question?

$$2(12x + 4) = 2(18x + \boxed{\phantom{00}})$$

A rectangle has perimeter 12 cm and width 4 cm.

An enlargement of that rectangle has perimeter 18 cm. What is its width?

David and Amber share money in the ratio 4 : 12.

David

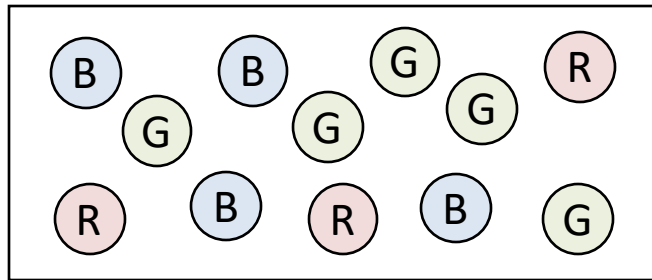


£

Amber



£18



Charlotte takes a counter from the box at random 18 times. Estimate the number of times she will take a blue counter.

$$12x = 4y$$

$y$  is equal to 18.  
What is the value of  $x$ ?

12% of a number is 4.

What is 18% of that number?

**(1)**

Which question is the odd one out? Why?

**(2)**

How could you change that question so that it would fit with the others?

**(3)**

What does 3 represent in each question?

**(8)**

*'Mary walked 4 miles in 12 hours. How many miles would she walk in 18 hours?'*

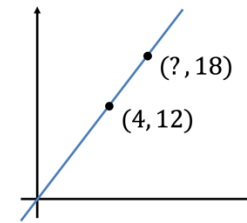
Can the other questions be adapted in the same way?

Proportion

$$\frac{4}{12} = \frac{?}{18}$$

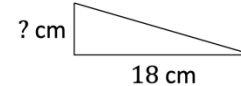
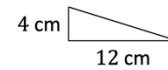
Kathryn earns £12 for every 4 trees she plants.

How much does she earn for planting 18 trees?



Mary walks 12 miles in 4 hours.

How long does it take her to walk 18 miles?



4 yards is equal to 12 feet.

How many yards are there in 18 feet?

Interweaving

**(4)**

How could the graph be used to represent each of the other questions?

**(7)**

What extra questions could be added that would fit with the others?

**(6)**

What does 1.5 represent in each question?

**(5)**

How could a ratio table be used for each question?

Which fractions equal 0.3 when rounded to 1 decimal place?

$\frac{1}{3}$	$\frac{35}{99}$	$\frac{25}{99}$
$\frac{1}{4}$	$\frac{35}{100}$	$\frac{25}{100}$
$\frac{1}{5}$	$\frac{35}{101}$	$\frac{25}{101}$

Find three fractions that add up to an integer.

They must all be different...

They must be proper fractions...

They must be fully simplified...

They must have a numerator of 1...

Fill the gaps using the digits from 1 to 9 at most once each.

(a)

$$\square \frac{\square}{\square} + \square \frac{\square}{\square} = \square \frac{\square}{\square}$$

(b)

$$\square \frac{\square}{\square} \times \square \frac{\square}{\square} = \square \frac{\square}{\square}$$

**Example:** Converting from a recurring decimal to a fraction

*Sam has answered this question **correctly**.*

**Q:** Using algebra, convert  $0.\dot{3}\dot{6}$  to a fraction.

**A:**  $x = 0.363636\dots$

$$100x = 36.363636\dots$$

$$99x = 36$$

$$x = \frac{36}{99} = \frac{4}{11}$$

*Your turn*

**Q:** Using algebra, convert  $0.\dot{5}\dot{7}$  to a fraction.

**A:**

**1:** Explain the benefit of Sam multiplying by 100.

**2:** Use Sam's answer to write the following as fractions

a)  $0.\dot{7}\dot{2}$

b)  $0.\dot{3}\dot{7}$

c)  $0.8\dot{6}\dot{3}$

**3:** Using algebra, convert  $0.\dot{9}$  to a fraction.

	Amount (A)	Percentage (P%)	P% of A	A increased by P%	A decreased by P%
a.	75	60 %			
b.	75		60		
c.	75				60
d.	60			75	
e.		60 %	75		
f.		60 %		75	
g.		60 %			75
h.			60	75	
i.			60		75
j.				75	60

# Percentages

	Amount (A)	Percentage (P%)	P% of A	A increased by P%	A decreased by P%
a.	75	60 %	45	120	30
b.	75	80 %	60	135	15
c.	75	20 %	15	90	60
d.	60	25 %	15	75	45
e.	125	60 %	75	200	50
f.	46.875	60 %	28.125	75	18.75
g.	187.5	60 %	112.5	300	75
h.	15	400 %	60	75	-45
i.	135	$44\frac{4}{9}$ %	60	195	75
j.	67.5	$11\frac{1}{9}$ %	7.5	75	60

	Amount (A)	Percentage (P%)	P% of A	A increased by P%	A decreased by P%
a.		64 %	377 856		
b.		64 %		377 856	
c.		64 %			377 856
d.		42 %			
e.		42 %			
f.		42 %			
g.			313 344		
h.				313 344	
i.					313 344

- Why was 377 856 chosen for rows a-c?
- What number could be used in rows d-f?
- What percentage could be used in rows g-h?

# Percentages

	Amount (A)	Percentage (P%)	P% of A	A increased by P%	A decreased by P%
a.	590 400	64 %	377 856	968 256	212 544
b.	230 400	64 %	147 456	377 856	82 944
c.	1 049 600	64 %	671 744	1 721 344	377 856
d.	823 600	42 %	345 912	1 169 512	477 688
e.	243 600	42 %	102 312	345 912	141 288
f.	596 400	42 %	250 488	846 888	345 912
g.	870 400	36 %	313 344	1 183 744	557 056
h.	230 400	36 %	82 944	313 344	147 456
i.	489 600	36 %	176 256	665 856	313 344

- Why was 377 856 chosen for rows a-c?
- What number could be used in rows d-f?
- What percentage could be used in rows g-h?

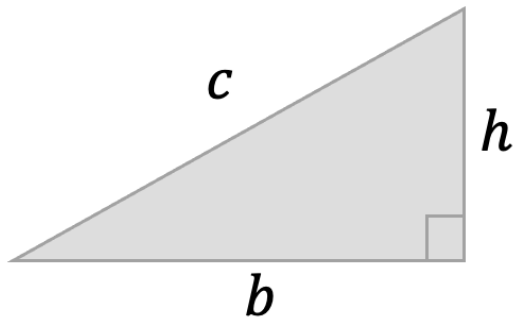
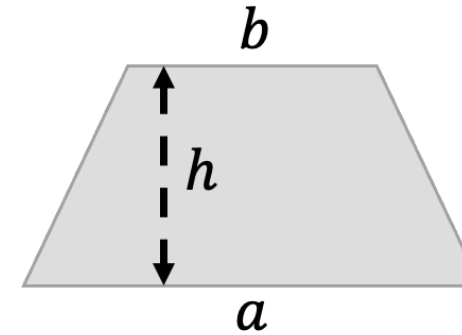
What was the original price of the car? (Non-calculator)



## Sharing in a Ratio – Area and Perimeter

A trapezium has area  $A$ , parallel sides  $a$  and  $b$ , and height  $h$ . Find:

1. The area, if  $h = 12$  cm and  $a : b : h = 5 : 4 : 3$ .
2. The height, if  $A : a : b = 540 : 2 : 3$  and  $b$  is 20 cm longer than  $a$ .

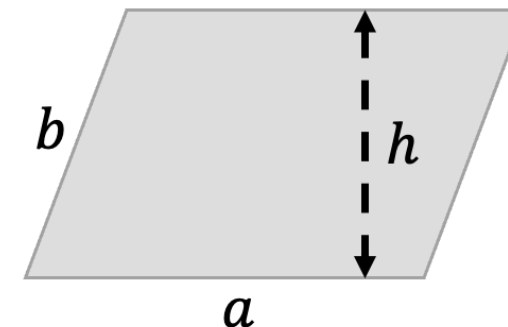


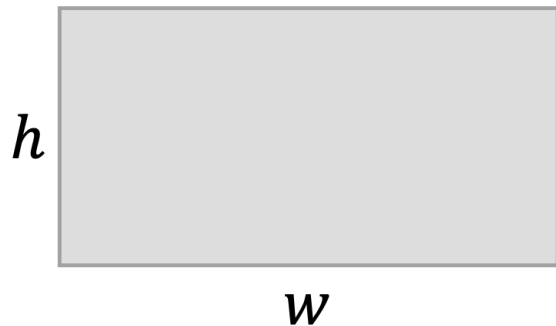
A right-angled triangle has area  $A$ , perimeter  $P$ , and sides  $b$ ,  $c$ , and  $h$ . Find:

3. The area, if the perimeter is 72 cm and  $b : c : h = 4 : 5 : 3$ .
4. The size of the smallest angle, if the angles are in the ratio  $6 : 19 : 25$ .

A parallelogram has area  $A$ , perimeter  $P$ , height  $h$ , and sides  $a$  and  $b$ . Find:

5. The ratio  $a : b : P$ , if the area is  $40 \text{ m}^2$  and  $a$  is 6 m longer than  $h$  and 5 m longer than  $b$ .
6. The value of  $h$ , if  $a : P : A = \frac{1}{12} : \frac{2}{3} : 0.18$





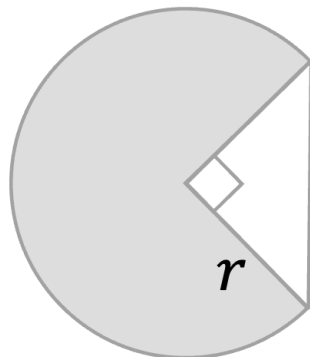
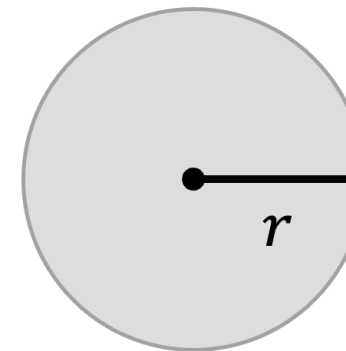
For a rectangle with width  $w$ , height  $h$ , area  $A$ , and perimeter  $P$  find:

7. The area, if  $P = 36$  cm and the ratio of  $h$  to  $w$  is  $2 : 1$ .
8. The perimeter, if the area is  $0.25 \text{ cm}^2$  and the ratio  $A : w = 1 : 3$ .  
Leave your answer as a mixed number.

A circle has area  $A \text{ m}^2$ , circumference  $C \text{ m}$ , and radius  $r \text{ m}$ . Find:

9. The radius, if the ratio  $r : A = 14 : 95$ .

10. The diameter, if the ratio  $C : A = 1 : 54$ .



The shaded shape is three-quarters of a circle with radius  $r$ .

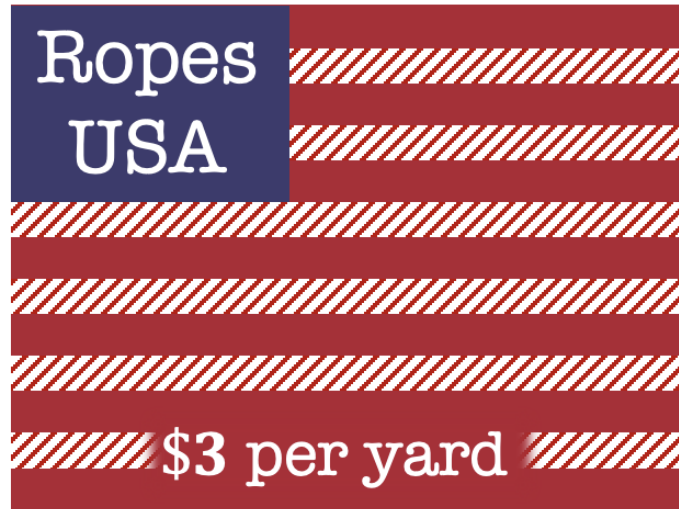
11. The ratio of the shape's perimeter to its radius, in terms of  $\pi$ .
12. The ratio of the area of the shaded shape to the area of the unshaded triangle.

Jo and Mo have many paperweights, in the ratio of **8 : 3**.

Jo, being a generous person, gives Mo **14** of her paperweights the ratio is now **6 : 5**.

How many paperweights did each have initially?

Here are adverts from two rival companies:  
Ropes USA and the EUROPEAN ROPE COMPANY



1 yard = 91.44 cm  
\$1 = €0.92

Which company is offering the better deal?

## Interwoven Maths



- Home
- Contents
- Interwoven Tasks
- Booklets
- Other

### Product Rule for Counting

e.g. How many factors does 180 000 have?  
 $180\,000 = 10\,000 \times 18$   
 $= 2^4 \times 5^4 \times 2 \times 3^2$   
 $= 2^5 \times 3^2 \times 5^4$   
 Number of factors =  $(5+1)(2+1)(4+1) = 6 \times 3 \times 5 = 90$

a) How many factors does 245 000 have?  
 b) How many odd factors does 245 000?  
 c) How many square numbers are factors of 245 000?  
 d) How many factors of 245 000 are also factors of 180 000?  
 e) How many factors of 180 000 are **not** also factors of 245 000?  
 f) Find three other numbers that would each have the same number of factors as 245 000.

(IW) Product Rule for Counting Factors

### Introducing Sequences And Series an interwoven approach

(IW) Introducing Geometric Series

interwovenmaths.com

#### Volume and Surface Area of a Cuboid

	A	B	C	Volume	Surface Area
a	1 cm	2 cm	10 cm		
b	1 cm	2 cm		10 cm <sup>3</sup>	
c	1 cm	2 cm			10 cm <sup>2</sup>
d	2 cm	5 cm	90 cm <sup>3</sup>		
e	2 cm	5 cm		90 cm <sup>3</sup>	
f	2 cm	5 cm			90 cm <sup>2</sup>
g	4 cm			400 cm <sup>3</sup>	400 cm <sup>2</sup>
h	4 cm			288 cm <sup>3</sup>	288 cm <sup>2</sup>
i	5 cm			250 cm <sup>3</sup>	250 cm <sup>2</sup>

(IW) Interwoven Volume and Surface Area Completion Tables

#### Prime numbers less than 100

Prime	Composite	Prime	Composite	Prime	Composite
2	4	11	16	22	28
3	6	13	18	23	29
5	10	17	20	29	35
7	14	19	24	31	36
11	22	23	28	37	42
13	26	29	34	41	46
17	34	31	38	43	48
19	38	37	44	47	52
23	46	41	48	49	54
29	56	43	50	53	58
31	60	47	54	59	62
37	68	53	60	61	66
41	74	59	66	67	70
43	78	61	70	71	76
47	84	67	76	73	80
53	90	71	80	79	86
59	96	73	84	83	90
61	100	79	90	89	96
67		83	96	97	100

(IW) Histograms and Cumulative Frequency

#### Fractions

Find the mean, median, and range of:  
 $3\frac{1}{2}$ ,  $6\frac{1}{6}$ , and  $2\frac{1}{2}$

#### Area and Perimeter

A rectangle has a width of 3 cm and a height of 2 cm.  
 Draw a second rectangle so that the two rectangles have a mean area of 13 cm<sup>2</sup> and have perimeters with a range of 8 cm.

(IW) Interwoven Revision Questions to Project

#### Standard Form

Find the median of the following:  
 $3 \times 10^{-4}$ ,  
 $4 \times 10^{-3}$ ,  
 $5 \times 10^{-6}$ ,  
 $6 \times 10^{-5}$ .

#### Averages with...

Find the upper bound for the median angle in a quadrilateral.  
 Is it possible to actually draw a quadrilateral with that median angle?

#### Angles in polygons with...

**Mean**  
 A regular polygon has interior angles that are 120° bigger than its exterior angles.  
 How many sides does it have?

**Median**  
 A regular polygon has interior angles that are 270° of the size of the angle of its exterior angles.  
 How many sides does it have?

**Range**  
 A regular polygon has interior angles that round to 120° to 2 significant figures.  
 How many sides could it have?

#### Circle Theorems

**Circle Theorems**  
 In the centre of the circle, all other points are on the circumference. All radii are the same length.

$AB = 20^\circ$   
 What is  $ACB$ ?

$ACB = 20^\circ$   
 What is  $AB$ ?

$AB = 20^\circ$   
 What is  $AC$ ?

$AB = 20^\circ$   
 What is  $AE$ ?

(IW) Angles in Polygons with... (Mixed Questions)

#### Circle Theorems

**Circle Theorems**  
 In the centre of the circle, all other points are on the circumference. All radii are the same length.

$ABC = 20^\circ$   
 What is  $ACB$ ?

$ABC = 20^\circ$   
 What is  $AB$ ?

$AB = 20^\circ$   
 What is  $AC$ ?

$AB = 20^\circ$   
 What is  $AE$ ?

(IW) Circle Theorems with... (no diagrams!)

#### Trigonometry with...

**Area**  
 A right-angled triangle has a hypotenuse of 10 cm and one of the other sides is 6 cm. Find the area of the triangle.

**Angles**  
 A right-angled triangle has a hypotenuse of 10 cm and one of the other sides is 6 cm. Find the angles of the triangle.

#### Pythagoras with...

**Area**  
 A right-angled triangle has a hypotenuse of 10 cm and one of the other sides is 6 cm. Find the area of the triangle.

**Angles**  
 A right-angled triangle has a hypotenuse of 10 cm and one of the other sides is 6 cm. Find the angles of the triangle.

#### Averages with...

**Mean**  
 The mean of 10 numbers is 12. One of the numbers is 20. What is the mean of the other 9 numbers?

**Median**  
 The median of 10 numbers is 12. One of the numbers is 20. What is the median of the other 9 numbers?

**Range**  
 The range of 10 numbers is 12. One of the numbers is 20. What is the range of the other 9 numbers?

**(1)** Which question is the odd one out? Why?

**(2)** How could you change that question so that it would fit with the others?

**(3)** What does 3 represent in each question?

**(4)** How could the graph be used to represent each of the other questions?

**(5)** How could a ratio table be used for each question?

**(6)** What does 1.5 represent in each question?

**(7)** What extra questions could be added that would fit with the others?

**(8)** Myra walked 6 miles in 12 hours. How many miles would she walk in 28 hours? Can the other questions be adapted in the same way?

**(9)** What extra questions could be added that would fit with the others?

#### Areas of Trapeziums with... Fractions and Decimals

Find the area of each trapezium.



**Nathan Day**  
 @nathanday314

Maths teacher.  
 Task interweaver.

📍 Nottingham, England 🌐 [interwovenmaths.com](https://interwovenmaths.com)  
 📅 Joined March 2015

367 Following 8,194 Followers

interwovenmaths.com  
 nathanday42@gmail.com  
 @nathanday314

# Thank you!