Core Pure 1 Revision Mat

1. Given $|z - 5 - 12\mathrm{i}| = 3$, find the least value of $|z|$.

2. A triangle of area 6 is mapped to a triangle of area 3 by the matrix. Find all values of θ, $0 \leq \theta < 180°$.

$$\begin{pmatrix} \cos 2\theta & 0 \\ 1 & \tan 2\theta \end{pmatrix}$$

3. Find, fully factorised,

$$\sum_{r=n+1}^{2n} r^2$$

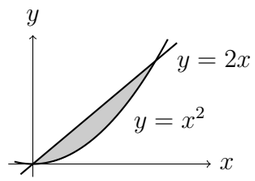
4. α, β, γ are the roots of $x^3 - 2x^2 + 5x - 4 = 0$. Find $\alpha^2 + \beta^2 + \gamma^2$.

5. Find the acute angle between the line and the plane.

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \qquad \mathbf{r} \cdot \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = 5$$

6. $f(n) = 5^n + 8n + 3$. Write $f(k+1) - 5f(k)$ in the form $a(bk+1)$.

7. The shaded region is rotated 2π about the x-axis. Find the volume.



8. Find the invariant lines through the origin for

$$\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$$

9. $z^3 - 4z^2 + pz + q = 0$, where $p, q \in \mathbb{R}$, has a root $z = 6$. On an Argand diagram the roots form a triangle of area 21. Find p and q.

10. Find the perpendicular distance from $(3, 1, -2)$ to the plane $2x - y + 2z = 10$.

11. $x^3 + 2x^2 - x + 3 = 0$ has roots α, β, γ. Find the cubic equation with roots $2\alpha + 1$, $2\beta + 1$, $2\gamma + 1$.

12. Find the values of k for which the matrix is singular.

$$\begin{pmatrix} 1 & k & 2 \\ k & 4 & 4 \\ 1 & 1 & 1 \end{pmatrix}$$

13. The half-line $\arg(z+1) = \theta$, $\theta > 0$, is a tangent to the circle $|z - 3| = 2$. Find the complex number z at the point of contact.

14. Find, fully factorised,

$$\sum_{r=5}^{n} (6r + 1)$$

15. The region bounded by the curve $4x^2 = y^2(9 - y^2)$, $x \geq 0$, $0 \leq y \leq 3$, and the y-axis is rotated 2π about the y-axis. Find the volume of the solid formed.

16. Find $(\mathbf{AB})^{-1}$ given

$$\mathbf{A}^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \qquad \mathbf{B}^{-1} = \begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$$

17. The lines intersect. Find p.

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} 1 \\ p \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

18. Solve $z^2 = 5 - 12\mathrm{i}$.

19. The matrix represents an enlargement, scale factor $k > 0$, and a rotation θ anticlockwise about O. Find k and θ.

$$\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$$

20. $x^4 - 3x^3 + 2x^2 + x - 5 = 0$ has roots $\alpha, \beta, \gamma, \delta$. Find

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$$

21. Find a in terms of n.

$$\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 2^n & a \\ 0 & 1 \end{pmatrix}$$

22. Find the area of the region

$$|z - 2\mathrm{i}| \leq 2, \qquad 0 \leq \arg z \leq \frac{\pi}{4}$$

23. Find the shortest distance from $(1, 2, 3)$ to the line

$$\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

24. The planes form a sheaf. Find b and c.

$$x + y - z = 2$$
$$2x - y + 3z = 1$$
$$x + by + 9z = c$$

25. $x^3 - 3x^2 + 4x + 2 = 0$ has roots α, β, γ. Find

$$(\alpha - 2)(\beta - 2)(\gamma - 2)$$

26. Find the image of the line $y = 2x + 1$ under

$$\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$$

27. Find n given

$$\sum_{r=1}^{n} r^3 = 78 \sum_{r=1}^{n} r$$

28. The region bounded by $y = x^2 + 1$, the x-axis, the y-axis and the line $x = 1$ is rotated 2π about the y-axis. Find the volume.

29. The roots of $x^3 - 9x^2 + kx - 15 = 0$ form an arithmetic sequence. Find k.

30. $\mathbf{M}$ is a rotation of 90° anticlockwise about the z-axis. $\mathbf{N}$ is a reflection in the plane $y = 0$. Find the image of $(-2, 4, 3)$ under $\mathbf{M}$ followed by $\mathbf{N}$.

31. $z = -\sqrt{3} + \mathrm{i}$ and $w = -1 - \mathrm{i}$. Find $\arg\left(\frac{z}{w}\right)$, where $-\pi < \arg \leq \pi$.

32. Find the reflection of the point $(1, 2, 3)$ in the plane $x + y + z = 0$.

33. $\mathbf{M}$ maps (x, y) to $(8, 11)$, where $x + y = 5$. Find a.

$$\mathbf{M} = \begin{pmatrix} a & 2 \\ 1 & 3 \end{pmatrix}$$

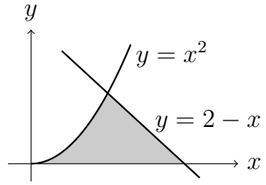
34. $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ are non-singular and $\mathbf{ABC} = \mathbf{I}$. Write $\mathbf{B}^{-1}$ in terms of $\mathbf{A}$ and $\mathbf{C}$.

35. Given $|z - 3 + \mathrm{i}| = |z - 1 - 5\mathrm{i}|$, find the least value of $|z|$.

36. The line is parallel to the plane $2x + ky - z = 5$. Find k.

$$\frac{x-1}{2} = \frac{3-y}{1} = \frac{z}{2}$$

37. The shaded region is rotated 2π about the x-axis. Find the volume.



38. Find the line of invariant points for

$$\begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix}$$

39. $x^3 - 2x^2 + 4 = 0$ has roots α, β, γ. Find $\alpha^3 + \beta^3 + \gamma^3$.

40. The roots of $z^3 - 3z^2 + z + 5 = 0$ form a triangle on an Argand diagram. Find its area.

41. Find $\mathbf{M}^{2027}$ given

$$\mathbf{M} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

42. Find, to 3 s.f., the standard deviation of the twenty numbers $3r + 7$, $r = 1, 2, \ldots, 20$.

43. $f(z) = z^4 + \ldots$ is a quartic with real coefficients. Two roots of $f(z) = 0$ are $-1 + 2\mathrm{i}$ and $1 + \mathrm{i}$. Find $f(0)$.

44. Find the shortest distance between the lines.

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

45. Find, to 3 s.f., the area of the region

$$|z - 3 - 4\mathrm{i}| \leq 5, \qquad 0 \leq \arg(z - 8\mathrm{i}) \leq \pi$$

46. $\mathbf{A}^2 = 2\mathbf{A} + 3\mathbf{I}$. Write $\mathbf{A}^{-1}$ in the form $p\mathbf{A} + q\mathbf{I}$.

47. The region bounded by $y = x + \frac{1}{x}$, the x-axis, $x = 1$ and $x = 2$ is rotated through θ radians about the x-axis. The volume is 3.625. Find θ.

48. Find w given

$$|w - 1 - \mathrm{i}| = \sqrt{10}, \qquad \arg(w - 2) = \frac{3\pi}{4}$$

49. Evaluate

$$\sum_{r=1}^{40} r \cos\left(\frac{r\pi}{2}\right)$$

50. $A(1, 2, 2)$ and $B(2, -1, 2)$. Find the exact area of triangle OAB.