Core Pure 1 - Vectors Practice

Question 1

The line l has equation

$$x - 3 = \frac{y-1}{2} = \frac{z+1}{2}$$

The plane Π has equation $2x - y + 2z = 15$

(a) Determine the coordinates of the point P where l meets Π. **(2)**

(b) Determine the acute angle between l and Π.
Give your answer in degrees to one decimal place. **(3)**

The point $A(3, 1, -1)$ lies on l.

(c) Determine the perpendicular distance from A to Π. **(2)**

The point $B(0, -5, -7)$ also lies on l.

(d) Deduce the perpendicular distance from B to Π. Explain your reasoning. **(2)**

Question 2

The plane Π_1 contains the points $A(1, 0, 1)$, $B(2, 2, 2)$ and $C(0, -1, 2)$.

(a) Write down an equation of Π_1 in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ **(2)**

(b) Determine a Cartesian equation of Π_1. **(5)**

The plane Π_2 has equation $x - 2y + 3z = 5$

(c) Determine the acute angle between Π_1 and Π_2.
Give your answer in degrees to one decimal place. **(3)**

The point $D(k, 2k, 5)$ lies on Π_1.

(d) Determine the value of k. **(1)**

Question 3

The plane Π has equation $x + 2y - 2z = 3$

The point A has coordinates $(3, 6, -3)$.

(a) Determine the coordinates of A', the image of A under reflection in Π. **(4)**

The point B has coordinates $(4, 3, -1)$.

(b) Show that A and B lie on the same side of Π. **(2)**

The point X lies on Π.

The total distance $AX + XB$ is as small as possible.

(c) Determine the coordinates of X and the exact minimum value of $AX + XB$. **(4)**

Question 4

The line l has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

The point A has coordinates $(7, 6, 2)$.

(a) Determine the shortest distance from A to l. **(5)**

(b) Determine the coordinates of the image of A under reflection in the line l. **(2)**

The points B and C lie on l.

$AB = AC = 3\sqrt{5}$

(c) Determine the coordinates of B and the coordinates of C. **(3)**

(d) Determine the area of triangle ABC. **(2)**

Question 5

The line l_1 has equation

$$x - 2 = \frac{y}{2} = \frac{z+2}{3}$$

The plane Π has equation $x + y + z = 6$

The line l_1 meets Π at the point P.

(a) Determine the coordinates of P. **(2)**

(b) Determine the acute angle between l_1 and Π.
Give your answer in degrees to one decimal place. **(3)**

Of all the lines that lie in Π and pass through P, the line l_2 is the one that makes the smallest angle with l_1.

(c) Determine a vector equation of l_2. **(5)**

Question 6

The line l has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$$

The plane Π has equation $2x - y + 2z = 9$

(a) Determine the coordinates of the point P where l meets Π. **(2)**

The line l' is the image of l under reflection in Π.

(b) Determine a vector equation of l'. **(6)**

(c) Show that l and l' are perpendicular. **(1)**

(d) Hence state the acute angle between l and Π, giving a reason for your answer. **(2)**

Question 7

The plane Π_1 has equation $x + 2y - z = 4$

The plane Π_2 has equation $2x - y + 3z = 3$

The planes Π_1 and Π_2 meet in the line l.

(a) Determine a vector equation of l. **(4)**

(b) Determine the acute angle between Π_1 and Π_2.
Give your answer in degrees to one decimal place. **(3)**

The plane Π_3 contains l and passes through the point $C(1, 3, 2)$.

(c) Determine a Cartesian equation of Π_3. **(4)**

The plane Π_4 has equation $x + 7y - 6z = k$, where k is a constant.

(d) Show that, for every value of k, the line l is either parallel to Π_4 or lies in Π_4. **(2)**

(e) Determine the value of k for which l lies in Π_4. **(1)**

Question 8

The line l has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

The plane Π has equation $x - y + z = 5$

(a) Show that l does not meet Π. **(2)**

(b) Determine the exact shortest distance between l and Π. **(3)**

(c) Determine a vector equation of the image of l under reflection in Π. **(3)**

The plane Π'

- is parallel to Π
- is not the same plane as Π
- is the same distance from l as Π is

(d) Determine a Cartesian equation of Π'. **(2)**

Question 9

The lines l_1 and l_2 have equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \qquad l_2 : \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

(a) Show that l_1 and l_2 are skew. **(3)**

(b) Determine the shortest distance between l_1 and l_2. **(6)**

(c) Determine a vector equation of the line that meets both l_1 and l_2 at right angles. **(2)**

The plane Π contains l_1 and is parallel to l_2.

(d) Determine a Cartesian equation of Π. **(2)**

(e) Explain why the distance from any point on l_2 to Π equals your answer to part **(b)**. **(1)**

Question 10

The line l_1 has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

The plane Π has equation $2x + y + 2z = 10$

The line l_1 meets Π at the point P.

(a) Determine the coordinates of P. **(2)**

(b) Show that the acute angle between l_1 and Π is 45°. **(2)**

The line l_2

- lies in Π
- passes through P
- is perpendicular to l_1

(c) Determine a vector equation of l_2. **(3)**

The line l_3

- lies in Π
- passes through P
- makes an angle of 45° with l_1

(d) Determine a vector equation of l_3. **(4)**

Two distinct lines lie in Π, pass through P, and make an angle of 60° with l_1.

(e) Determine a vector equation of each of these two lines. **(6)**

(f) Explain why no line in Π through P makes an angle of 30° with l_1. **(1)**

Question 11

The point A has coordinates $(3, 1, 2)$.

The point B has coordinates $(-1, 3, 6)$.

The points X for which $AX = BX$ lie on a plane Π_1.

(a) By writing X as (x, y, z), determine a Cartesian equation of Π_1. **(4)**

(b) Show that Π_1 passes through the midpoint of AB and is perpendicular to AB. **(2)**

The points X for which $AX^2 - BX^2 = 36$ lie on a plane Π_2.

(c) Determine a Cartesian equation of Π_2. **(2)**

(d) By considering triangle ABX for a point X on Π_2, give a geometric explanation for the value 36 in the equation $AX^2 - BX^2 = 36$ **(3)**

The line l has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$$

The point C lies on l and $AC = BC$.

(e) Determine the coordinates of C and the exact value of AC. **(3)**

Question 12

The lines l_1 and l_2 are skew and have equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \qquad l_2 : \mathbf{r} = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

The point A has coordinates $(2, 1, 0)$.

The plane Π_1 contains A and l_1.

(a) Determine a Cartesian equation of Π_1. **(4)**

The line l_3 passes through A and meets both l_1 and l_2.

(b) Determine a vector equation of l_3. **(4)**

(c) Determine the coordinates of the points where l_3 meets l_1 and l_2. **(3)**

The point B has coordinates $(4, 1, 0)$.

(d) Show that there is no line through B that meets both l_1 and l_2. **(4)**